

Night Vision and Electronic Sensors Directorate

RDER-NV-TR-267

A Note on the Brewster Angle in Lossy Dielectric Media

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A Note on the Brewster Angle in Lossy Dielectric Media

In electromagnetism, the Brewster angle, θ_B , is the angle of incidence at which there is no reflection of the parallel (i.e. vertical) polarized wave from a planar interface between materials with differing permittivity. That is, the reflection coefficient goes to zero at this angle and all of the incident power is transmitted into the dielectric medium. The reflection coefficient is a function of the constitutive parameters permittivity, ϵ , and permeability, μ , of the two media forming the interface as well as the angle of incidence. For media with $\mu_{rt} = \mu_{ri}$, the Brewster angle is commonly expressed as

$$\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_{rt}}{\epsilon_{ri}}}, \quad (1)$$

where the r subscript denotes the *relative* constitutive parameter, the i subscript denotes the parameter of the medium in which the *incident* wave is propagating, and the t subscript denotes parameter of the medium into which the wave is *transmitted*. We assume the incident medium is free space and the transmitted medium is soil for the purposes of ground penetrating radar (GPR). For perpendicularly polarized waves, no real incident angle exists that will reduce the reflection coefficient to zero. [1]

For clarity, we now define some basic definitions associated with plane waves incident at a planar interface:

1. Plane of incidence – the plane containing the propagation vector of the incident wave k_i and the unit normal to the interface.
2. Parallel polarization – the electric field of the incident wave lies in the plane of incidence. The orthogonal magnetic field is in the direction transverse to plane of incidence, so we call this a transverse magnetic (TM) wave. This is referred to as *vertical polarization* in the GPR community.
3. Perpendicular polarization – the electric field of the incident wave lies normal to the plane of incidence. Since the electric field is in the direction transverse to the plane of incidence, we call this a transverse electric (TE) wave. This is referred to as *horizontal polarization* in the GPR community.

In realistic environments involving GPR, one must consider the lossy component of the soil's permittivity. Soils become lossy as moisture is added, especially in mineralized soils with high salinity [2], [3]. The soil moisture strongly affects the real part of the permittivity. While the salinity of soils does not have a dramatic influence on the real part of the permittivity, the

imaginary part is strongly affected by both the salinity and moisture of the soil, especially at low frequencies.

Expressing the complex permittivity for the incident wave medium and the transmitted wave medium as

$$\tilde{\epsilon}_{ri} = \epsilon'_{ri} - j\epsilon''_{ri}, \quad (2)$$

$$\tilde{\epsilon}_{rt} = \epsilon'_{rt} - j\epsilon''_{rt}, \quad (3)$$

we can attempt to find the Brewster angle using (1) as follows. We change the complex permittivity into phasor form to make the mathematics associated with the square root slightly easier. That is, we can write

$$\theta_B = \tan^{-1} \frac{\epsilon'_{rt} - j\epsilon''_{rt}}{\epsilon'_{ri} - j\epsilon''_{ri}} = \tan^{-1} \frac{|\tilde{\epsilon}_{rt}| \exp(j\phi_{rt})}{|\tilde{\epsilon}_{ri}| \exp(j\phi_{ri})} = \tan^{-1} \left(\frac{|\tilde{\epsilon}_{rt}|}{|\tilde{\epsilon}_{ri}|} \exp \left[j \frac{1}{2} (\phi_{rt} - \phi_{ri}) \right] \right). \quad (4)$$

where we use ϕ to represent the phase angle as opposed to the azimuthal angle of incidence. Now, changing back from phasor form in order to take the inverse tangent, the argument becomes

$$\theta_B = \tan^{-1} \left[\underbrace{\sqrt{\frac{|\tilde{\epsilon}_{rt}|}{|\tilde{\epsilon}_{ri}|}} \cos \left(\frac{1}{2} [\phi_{rt} - \phi_{ri}] \right)}_a + j \underbrace{\sqrt{\frac{|\tilde{\epsilon}_{rt}|}{|\tilde{\epsilon}_{ri}|}} \sin \left(\frac{1}{2} [\phi_{rt} - \phi_{ri}] \right)}_b \right]. \quad (5)$$

We then attempt to solve for the Brewster angle using the definition of the inverse tangent of a complex number as

$$\theta_B = \tan^{-1}(a + jb) = \frac{j}{2} \ln \left[\frac{1 - j(a + jb)}{1 + j(a + jb)} \right], \quad (\text{not a realizable angle!}) \quad (6)$$

where the constants a and b are defined in (5) and \ln is the natural logarithm.

It can be seen from (6) that the Brewster angle for a complex valued permittivity is a complex value. Since complex valued physical angles do not exist, we can conclude that the reflection coefficient cannot truly go to zero in a soil with complex valued permittivity. In other words, in a lossy soil there is no angle for which all of the incident TM wave power will be transmitted into the soil.

Though the reflection coefficient does not exactly go to zero in a lossy medium, there still exists a minimum value at some angle, which is often referred to as the pseudo-Brewster angle. We define the reflection coefficient for a TM wave incident upon a soil from free space as

$$\Gamma^{TM} = \frac{\tilde{n}^2 \sin \theta_i - \sqrt{\tilde{n}^2 - \cos^2 \theta_i}}{\tilde{n}^2 \sin \theta_i + \sqrt{\tilde{n}^2 - \cos^2 \theta_i}}, \quad (7)$$

where θ_i is the incident angle and \tilde{n} is the complex index of refraction of the soil

$$\tilde{n} = \sqrt{(\varepsilon'_r - j\varepsilon''_r)(\mu_r)}. \quad (8)$$

If we assume a nonmagnetic medium, the relative permeability in (8) will be unity, i.e. $\mu_r = 1$. We can observe how the imaginary component of the permittivity affects the wave propagation at a hypothetical interface by plotting the reflection coefficient for several values of imaginary permittivity while keeping the real part constant. Figure 1 shows the magnitude of the reflection coefficient plotted versus angle of incidence for four different imaginary permittivity components. The real component of the permittivity was kept constant at unity. For the case of no loss and a very low (unity) real permittivity, we observe no reflection. For the lossy case, we can see that the imaginary component of the permittivity affects the angle at which we find the dip in the reflection coefficient. We can also observe that this dip becomes sharper for higher losses. We can draw two important conclusions from these observations:

1. One cannot simply use the real part of the permittivity to determine the angle for minimum reflection (which we may call the “pseudo-Brewster angle”) given a lossy soil.
2. In real environments the soil surface (dielectric interface) is not perfectly flat. If there is a large amount of loss in the soil, small deviations in angle around the pseudo-Brewster angle will significantly increase the reflection coefficient.

As an example demonstrating the second point stated above, the change in the reflection coefficient due to angle deviations around the pseudo-Brewster angle can be observed in Figure 1 for the highest loss case. The minimum value of the reflection coefficient for the case where $\tilde{\varepsilon}_r = 1 - j100$ occurs when $|\Gamma|^2 = 0.17$ at $\theta_i = 84.25^\circ$. The reflection coefficient doubles if the angle of incidence is reduced by -8.75° or if it is increased by only 3.5° . This change in

reflection due to small deviations in angle would preclude the exploitation of the Brewster angle for GPR applications in lossy soils with realistic undulating surfaces.

However, if we examine the case of a medium with low loss, say $\tilde{\epsilon}_r = 1 - j$, the reflection coefficient doubles from its minimum if the angle is reduced by -16.85° or if it is increased by 10.05° . In the low loss case, there is slightly more leeway for angle deviations while keeping the reflection coefficient relatively low.

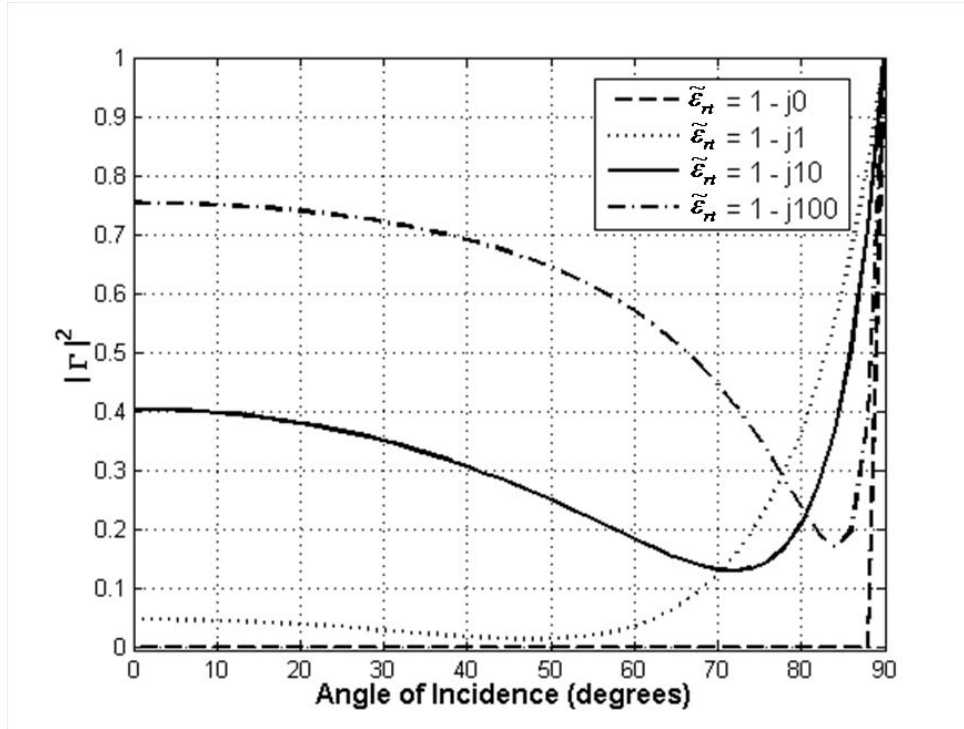


Figure 1. Reflection coefficient for media of varying levels of loss.

When moisture is added to soil, the real part of the permittivity is affected as well as the imaginary component. Using data collected from the U.S. Army RDECOM CERDEC Night Vision and Electronic Sensors Directorate's mine lanes facility and reported in [4], the reflection coefficient can be determined for a soil with varying levels of moisture. The soil under test in the present analysis is called bluestone gravel. The complex permittivity was derived using S-parameter measurements of a soil sample in a coaxial holder from 1 MHz – 2 GHz. Varying levels of moisture were added to the sample and the volumetric moisture percentage was recorded. Table 1 shows the permittivity values reported from the measurements at 200 MHz. Figure 2 shows the reflection coefficient for the bluestone gravel versus incident angle as calculated using equation (7) for a frequency of 200 MHz.

Table 1. Permittivity values of “bluestone gravel” soil from the NVESD mine lanes facility at 200 MHz.

Measured Permittivity for Bluestone Gravel at 200 MHz	
Volumetric Moisture (%)	Permittivity ($\tilde{\epsilon}_r = \epsilon'_r - j\epsilon''_r$)
0.8	$3.0 - j0.2$
2.5	$3.6 - j0.7$
8.3	$5.1 - j2.7$
15.8	$8.4 - j8.0$
30.3	$19.0 - j17.0$

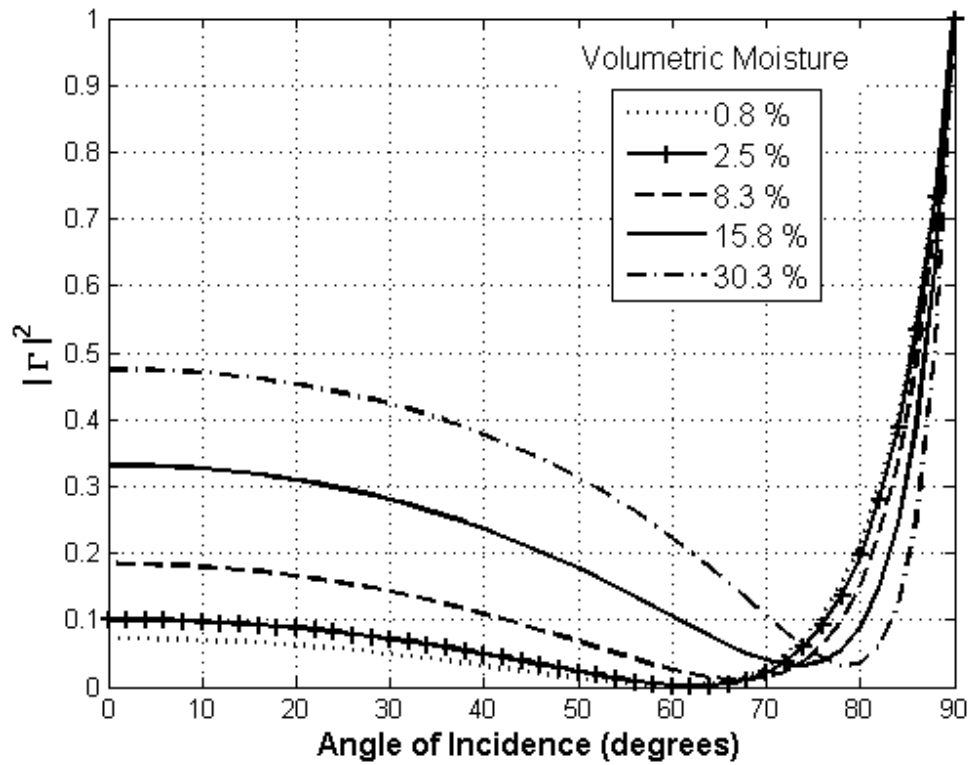


Figure 2. Reflection coefficient for soil from the NVESD mine lanes facility for varying levels of moisture at 200 MHz.

Table 2 shows the permittivity values reported from measurements at 2 GHz. Figure 3 shows the reflection coefficient for the bluestone gravel versus incident angle as calculated using equation (7) for a frequency of 2 GHz. While the reflection coefficient reaches a minimum very close to zero in this case, it should be noted that this minimum changes drastically with changes in volumetric moisture.

Table 2. Permittivity values of “bluestone gravel” soil from the NVESD mine lanes facility at 2 GHz.

Measured Permittivity for Bluestone Gravel at 2 GHz	
Volumetric Moisture (%)	Permittivity ($\tilde{\epsilon}_r = \epsilon'_r - j\epsilon''_r$)
0.8	$2.8 - j0.0$
2.5	$3.1 - j0.1$
8.3	$4.4 - j0.5$
15.8	$7.5 - j1.4$
30.3	$17.5 - j3.3$

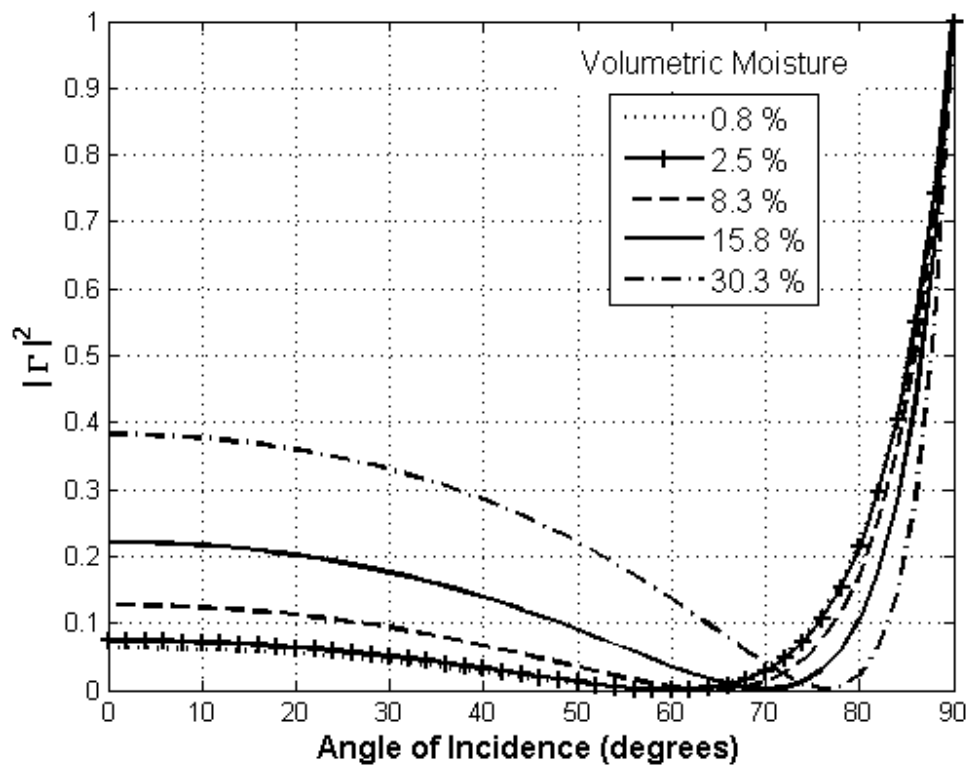


Figure 3. Reflection coefficient for soil from the NVESD mine lanes facility for varying levels of moisture at 2 GHz.

The Brewster angle expression given in (6) merely shows that there is no physical angle at which the reflectivity goes to zero for a lossy medium and should not be used for actual calculations. One may be tempted to take the real part or the absolute value of the complex angle, but neither of these corresponds to the minimum value of the reflection coefficient. It can be shown that the reflection coefficient reaches a minimum when the real part of $\Gamma = 0$. As stated in [5] without derivation, the pseudo-Brewster angle, which is *the angle for which the absolute magnitude of the complex reflection coefficient reaches its minimum*, $\tilde{\theta}_B$, is given as

$$\sin^2 \tilde{\theta}_B = \frac{(\varepsilon'_{rt} - 1) + \sqrt{(\varepsilon'_{rt}{}^2 + \varepsilon''_{rt}{}^2)(\varepsilon'_{rt} - 1)^2 + \varepsilon''_{rt}{}^2[(\varepsilon'_{rt}{}^2 + \varepsilon''_{rt}{}^2) - 1]}}{(\varepsilon'_{rt}{}^2 + \varepsilon''_{rt}{}^2) - 1}, \quad (9)$$

for $\varepsilon'_{ri} = 1$ and $\varepsilon''_{ri} = 0$ as in the case where the incident field is in free space. The angle given in (9) is the angle from grazing. Equation (9) can be verified by examining the reflection coefficients plotted in Figure 2. For example, the calculated minimum reflection coefficient at 200 MHz for 30.3% volumetric moisture ($\tilde{\varepsilon}_{rt} = 19 - j17$) is $|\Gamma|_{\min}^2 = 78.74^\circ$, which is in accordance with the plotted value. The calculated minimum reflection coefficient at 2 GHz for 30.3% volumetric moisture ($\tilde{\varepsilon}_{rt} = 17.5 - j3.3$) is $|\Gamma|_{\min}^2 = 76.66^\circ$. This example not only shows the validity of equation (9), it also shows that dispersion plays a role in determining the pseudo-Brewster angle. Since the complex permittivity is a function of frequency, the reflection coefficient must be a function of frequency too. Most GPR systems are wideband, which means that the pseudo-Brewster angle will not be constant over the entire radiated band.

It has been shown in this note that the Brewster angle does not exist for dielectric interfaces involving complex permittivity. A pseudo-Brewster angle does exist, which is the angle at which the magnitude of the reflection coefficient reaches a minimum. However, the pseudo-Brewster angle cannot be calculated using the standard Brewster angle formula. Furthermore, the reflection coefficient can rapidly rise for small angle deviations away from the pseudo-Brewster angle when the soil is very lossy. Finally, the dispersive nature of lossy soils means that the pseudo-Brewster angle will not be constant over a broad band.

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Appendix

Derivation for the pseudo-Brewster angle for media with complex valued permittivity.

The reflection coefficient for a parallel polarized wave incident from free space onto a nonmagnetic medium is given as

$$\Gamma^{TM} = \frac{n^2 \sin \theta_i - \sqrt{n^2 - \cos^2 \theta_i}}{n^2 \sin \theta_i + \sqrt{n^2 - \cos^2 \theta_i}}. \quad (A1)$$

where $n = \sqrt{\epsilon'_r - j\epsilon''_r}$ for nonmagnetic media. With complex valued permittivity and nonmagnetic media, we can rewrite (A1) as

$$\Gamma^{TM} = \frac{(\epsilon'_r - j\epsilon''_r) \sin \theta_i - \sqrt{(\epsilon'_r - j\epsilon''_r) - \cos^2 \theta_i}}{(\epsilon'_r - j\epsilon''_r) \sin \theta_i + \sqrt{(\epsilon'_r - j\epsilon''_r) - \cos^2 \theta_i}}. \quad (A2)$$

Each square root can be separated into a complex number using the following identity:

$$\sqrt{x + jy} = \sqrt{\frac{\sqrt{x^2 + y^2} + x}{2}} \pm j\sqrt{\frac{\sqrt{x^2 + y^2} - x}{2}}, \quad (A3)$$

where the sign should be chosen to be the same as the sign of y . After separating the square roots, we can combine real and imaginary terms and then multiply the top and bottom by the conjugate of the denominator to get the following

$$\Gamma^{TM} = \frac{\left[(\epsilon'^2_r + \epsilon''^2_r) \sin^2 \theta_i - \sqrt{(\epsilon'_r - \cos^2 \theta_i)^2 + \epsilon''^2_r} \right] + j[2 \sin \theta_i (A' - A'')]}{(\epsilon'^2_r + \epsilon''^2_r) \sin^2 \theta_i + 2 \sin \theta_i (A' + A'') + \sqrt{(\epsilon'_r - \cos^2 \theta_i)^2 + \epsilon''^2_r}}, \quad (A4)$$

$$A' = \epsilon'_r \sqrt{\frac{\sqrt{(\epsilon'_r - \cos^2 \theta_i)^2 + \epsilon''^2_r} - (\epsilon'_r - \cos^2 \theta_i)}{2}} \quad (A5)$$

$$A'' = \epsilon''_r \sqrt{\frac{\sqrt{(\epsilon'_r - \cos^2 \theta_i)^2 + \epsilon''^2_r} + (\epsilon'_r - \cos^2 \theta_i)}{2}} \quad (A6)$$

The pseudo-Brewster angle for lossy media $\tilde{\theta}_B$, which we are defining as *the angle for which the magnitude of the complex reflection coefficient reaches its minimum*, occurs when the real part of the complex reflection coefficient equals zero. Therefore, to solve for the Brewster angle, we simply set the real part of (A4) equal to zero. The denominator of the real part of (A4) vanishes when solving the algebraic equation so that we only need to solve the following equation for θ_i

$$(\varepsilon_r'^2 + \varepsilon_r''^2) \sin^2 \theta_i - \sqrt{(\varepsilon_r' - \cos^2 \theta_i)^2 + \varepsilon_r''^2} = 0. \quad (\text{A7})$$

Using the double angle formula and combining like cosine terms, (A7) takes the form

$$\cos^2 2\theta_i \underbrace{\left[(\varepsilon_r'^2 + \varepsilon_r''^2)^2 - 1 \right]}_a + \cos 2\theta_i \underbrace{\left[4\varepsilon_r' - 2(\varepsilon_r'^2 + \varepsilon_r''^2)^2 - 2 \right]}_b + \underbrace{\left[(\varepsilon_r'^2 + \varepsilon_r''^2)^2 - 4(\varepsilon_r'^2 + \varepsilon_r''^2) + 4\varepsilon_r' - 1 \right]}_c = 0 \quad (\text{A8})$$

We then use the quadratic equation

$$\cos 2\theta_i = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad (\text{A9})$$

where the constants are defined as shown in equation (A8). We choose the root with the negative sign in the numerator to assure that the quotient is between -1 and 1 and corresponds to real incident angles. The final step is to take the inverse cosine and solve for θ_i , which satisfies the condition for $\tilde{\theta}_B$, as follows

$$\tilde{\theta}_B = \frac{1}{2} \cos^{-1} \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right). \quad (\text{A10})$$

The expression given in (A10) can be condensed by assigning

$$E = \varepsilon_r'^2 + \varepsilon_r''^2, \quad (\text{A11})$$

which simplifies the pseudo-Brewster angle to

$$\tilde{\theta}_B = \frac{1}{2} \cos^{-1} \left(\frac{(1 + E^2 - 2\varepsilon_r') - \sqrt{(2\varepsilon_r' - E^2 - 1)^2 - (E^2 - 1)(E^2 - 4E + 4\varepsilon_r' - 1)}}{(E^2 - 1)} \right). \quad (\text{A12})$$

The solution (A11) can be shown to be equivalent to that of (9), which was given in the literature.

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